


Progression in Calculation Strategy

June 2015

CHARLESWORTH SCHOOL
...from tiny acorns great oaks grow

What you need to know about calculations

Mathematics will be at the core of your child's schooling from the moment they start to the moment they leave. They will be involved in drawing, measuring, statistics and lots of other practical activities that will help your child to understand and enjoy the subject.

This booklet offers guidance to the methods used to help our pupils with calculations. The methods we are advocating are in line with the revised National Curriculum (introduced in September 2014). We hope this will be helpful to you and that you will be able to support your child in learning by heart the basic rules which will assist in mental recall eg. number bonds and multiplication tables.

The methods that we use in school may or may not be familiar to you. Children are often confused when they ask parents for help at home and they try to teach the methods that they themselves were taught. Knowing how the methods in this booklet work will help you to help your children. All staff in school follow the methods outlined in this document so that we can ensure the consistency of our approach and can make sure that the children move onto the next step when they are ready.

The four operations that are covered by this booklet are addition, subtraction, multiplication and division. Whichever operation is being taught the child needs to experience all of these steps to completely conquer it.

- Using objects
- Using pictures
- Using a number line
- Using an expanded method
- Using a compact written method

Mental Calculation Strategies

Children should always be encouraged to consider if a mental calculation would be appropriate before using written methods. These are covered in the first part of each section.

Written Calculation Strategies

Why do children need to do written calculations?

- To represent work that has been done practically.
- To support, record and explain mental calculation
- To keep track of steps in a longer task
- To work out calculations that are too difficult to do mentally

Children should be taught when it is appropriate to do an approximate or estimate first and should check with the inverse operation at the end. By upper Key Stage 2, children should be confident in choosing and using a strategy that they know will get them to the correct answer as efficiently as possible.

Children will have specific lessons in each operation every term and enable the children to make rapid progress through the steps in each operation.

What can parents do to help?

- Count with their child
- Play number games
- Involve children when taking measurements or weighing items
- Take note of numbers in real life e.g. telephone numbers, bus numbers, lottery numbers etc.
- Give children opportunities to use money to shop, check change etc.
- Talking about the mathematics in football e.g. 'How many points does your favourite team need to catch the next team in the league?'

- When helping their children calculate use the method that they have been taught

Please don't…

- Teach your children that to multiply by 10 you 'just add a zero'. – you 'move the digits to the left and add a zero as a place holder'
- Tell them that you can move the decimal point. You can only move the digits to the left or to the right while the decimal point remains in the same place.
- Tell them that they are doing 'sums'. Language and vocabulary is important in Mathematics and 'sum' is a mathematical word that means 'addition', everything else is a 'calculation'
- Use the word 'number' to talk about a digit in a number. A number is a combination of digits that hold a certain value. For example in the 'number' 327, the '2' is referred to as the 'digit 2' with a value of two tens or 20.

Glossary of Terms

2-digit – a number with 2 digits like 23, 45, 12 or 60
3-digit – a number with 3 digits like 123, 542, 903 or 561
Addition facts – knowing that 1+1 = 2 and 1+3 = 4 and 2+5 = 7. Normally we only talk about number facts with totals of 20 and under.
Array -An array is an arrangement of a set of numbers or objects in rows and columns –it is mostly used to show how you can group objects for repeated addition or subtraction.
Bridge to ten – a strategy when using number lines. Adding a number that takes you to the next 'tens' number.
Short Division - traditional method for division with a single digit divisor
Concrete apparatus – objects to help children count – these are most often cubes (multilink) but can be anything they can hold and move. Dienes (purple hundreds, tens and units blocks), Numicon and Numicon rods are also referred to as concrete apparatus.
Column chunking – method of division involving taking chunks or groups or the divisor away from the larger number
Decimal number – a number with a decimal point
Divisor – the number you divide by in a division calculation. The number in each group for chunking.
Double – multiply a number by 2
Exchanging – Moving a 'ten' or a 'hundred' from its column into the next column and splitting it up into ten 'ones' (or 'units') or ten 'tens' and putting it into a different column
Expanded Multiplication – a method for multiplication where each stage is written down and then added up at the end in a column
Find the difference – A method for subtraction involving counting up from the smaller to the larger number
Grid method – a method for multiplying two numbers together involving partitioning
Half - a number, shape or quantity divided into 2 equal parts
Halve – divide a number by 2
Integer - a number with no decimal point
Inverse – the opposite operation. Addition is the inverse of subtraction, multiplication is the inverse of division
Long Multiplication – column multiplication where only the significant figures are noted
Number bonds to ten – 2 numbers that add together to make ten, like 2 and 8, or 6 and 4.
Number bonds to 100 – 2 numbers that add together to make 100 like 20 and 80, or 45 and 65 or 12 and 88
Number line – a line either with numbers or without (a blank number line). Children use this tool to help them count on for addition of subtraction and also in multiplication and division.
Number line Chunking - method of division involving taking chunks or groups or the divisor away from the larger number
Number sentence – writing out a calculation with just the numbers in a line E.G. 2+4=6 or 35 ÷7 = 5 or 12 x 3 =36 or 32 – 5 = 27
Partition – split up a larger number into the hundreds, tens and units. E.G. 342 – 300 and 40 and 2
Place Value – knowing that in the number 342 – the '3' means '3 hundreds', the '4' means '4 tens' and the '2' means '2'.
Quarter - a number, shape or quantity divided into 4 equal parts
Recombine – for addition, once you have partitioned numbers into hundreds, tens and units then you have to add then hundreds together, then add the tens to that total, then add the units to that total
Remainder – a whole number left over after a division calculation
Repeated addition – repeatedly adding groups of the same size for multiplication
Significant digit – the digit in a number with the largest value. E.G in 34 – the most significant digit is the 3, as it has a value of '30' and the '4' only has a value of '4'
Single digit – a number with only one digit. These are always less than 10.
Taking away – a method for subtraction involving counting backwards from the larger to the smaller number
Tens number - a number in the ten times tables – 10,20,30,40 50,etc.
Unit – another term for single digit numbers. The right hand column in column methods is the 'units' column

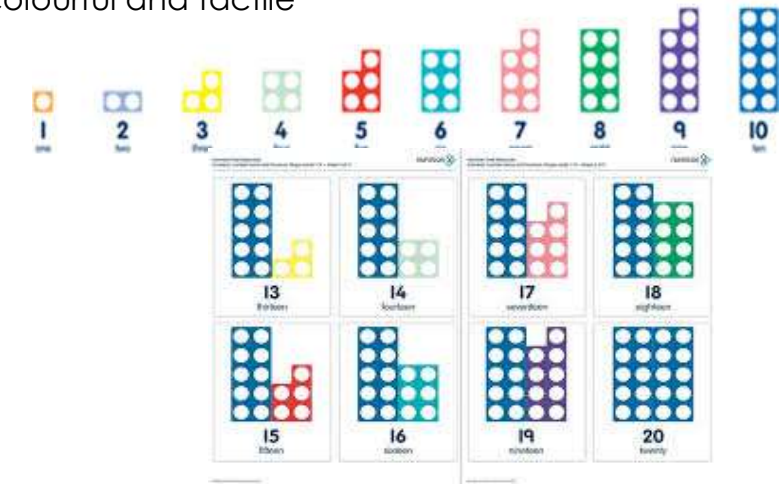
Practical Resources to Help Children Visualise Numbers and Calculation Methods

Base 10

Although it has been used in schools for many years, Base 10 is a crucial step in knowing what a 'one' (unit), a ten, a hundred and a thousand look like. It enables children to visualise their relative sizes and how they can be added together and split up to form smaller and larger numbers.

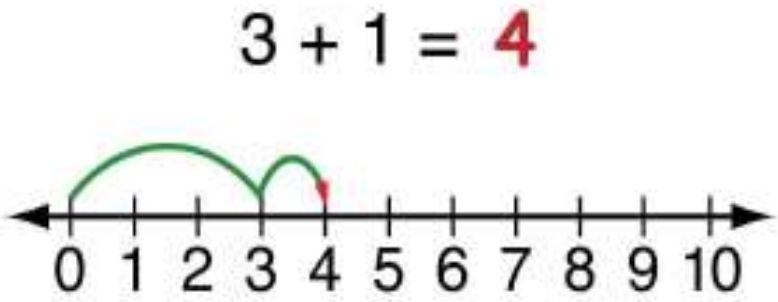
Numicon

Numicon is an especially useful resource as it can be used for teaching all four operations as well as fractions, decimals, percentages and a range of other aspects of maths. Each piece represents an integer from 1 to 10. The children love using it as it is colourful and tactile



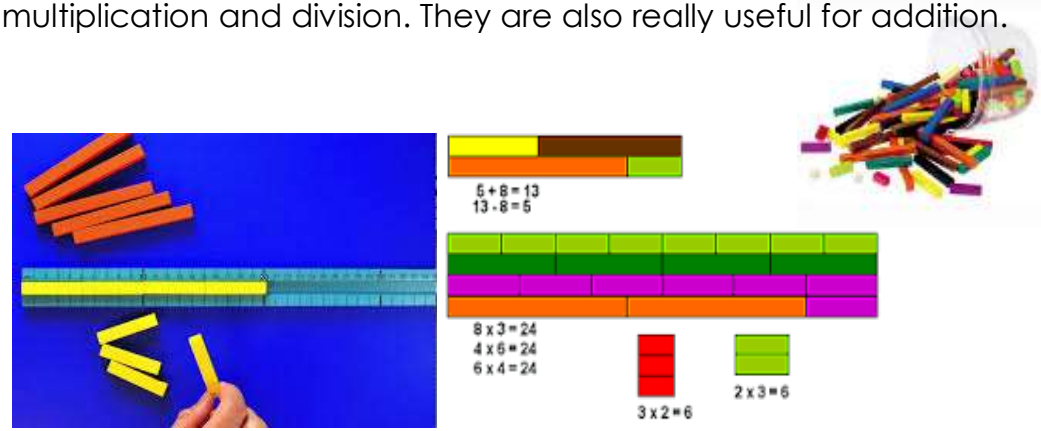
Number Lines

Number lines are a mainstay of teaching calculations. We have pre numbered and blank number lines in school that children can write on, or they can draw their own as appropriate for the calculation.



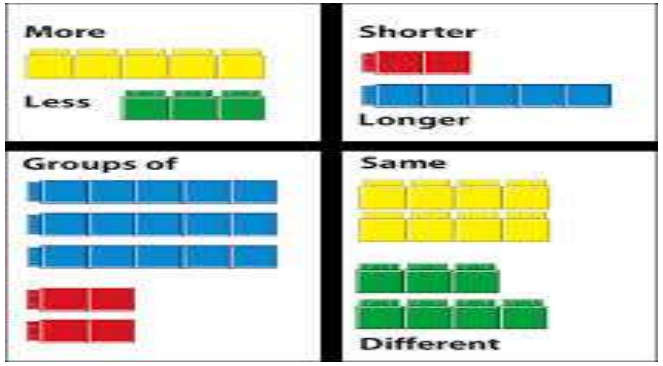
Numicon Rods and Number Tracks

Although these little rods that represent integers from 1 to 10, can be used for a range of aspects of maths, we normally use them for multiplication and division. They are also really useful for addition.



Unifix

Unifix are basic building blocks that help children to visualise differences in amounts and to add together multiples and share out amounts into equal arrays.



Number Fans

Number fans enable children to rehearse their answers to mental maths questions and reinforcing the place of each digit in the number.

Number Squares

Number squares help children to see the effect of adding or subtracting ones and tens by showing that you move either up and down the grid or forward and backwards. They also allow children to see patterns e.g., even and odd numbers

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Tables Squares

Tables squares help children to recall their times table and help them to see patterns such as square numbers

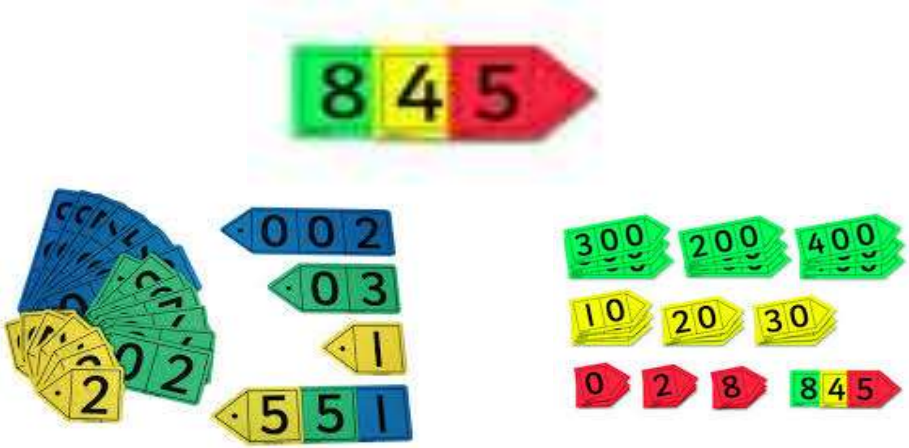
×	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

Place Value Counters

Place Value counters are discs that represent different place values either whole integers (1's, 10's, 100's etc) or decimal places (1/10's, 1/100's etc). Children can use these to help them understand the concept of carrying digits and exchanging. They can also use them to physically represent a digit in written methods.

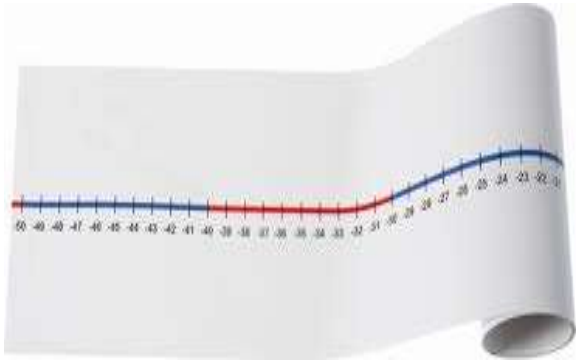
Arrow Cards (Place Value Cards)

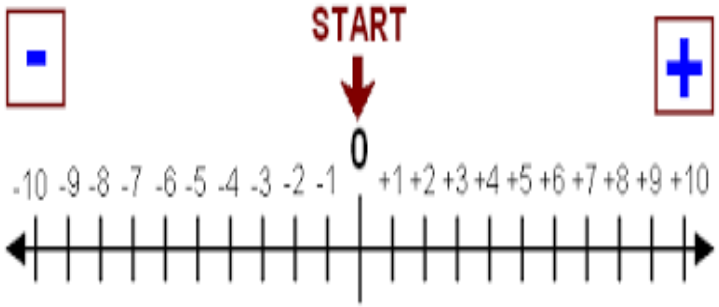
Arrow cards can be used to organise digits in a number so that the place and value of each digit is emphasised. They can also be used to support column methods by organising the digits in the correct column.

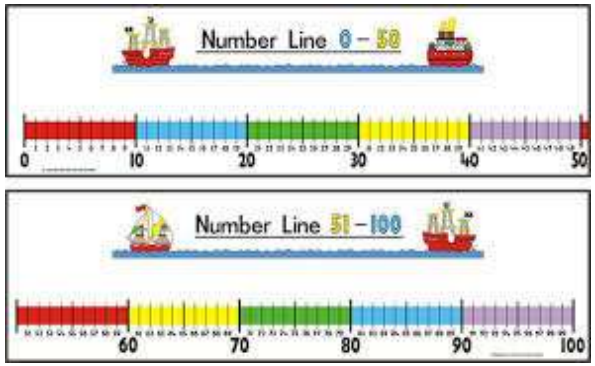


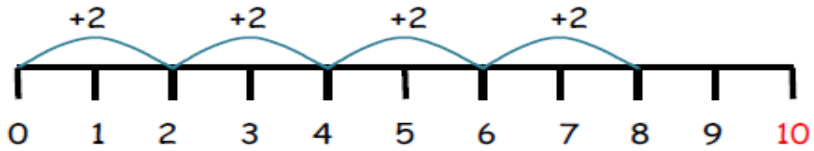
Number Lines

Number Lines are one of the most important tools for calculation. They come in a number of forms and can be used with decimal numbers and negative numbers. They help the children to count on and back in any number of jumps to support all four operations. Eventually, the pupils are encouraged to use a blank number line and to add the digits themselves. Pupils need to understand that the number line is a continuum – it has no 'official' starting or ending point.



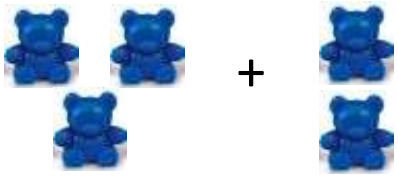




<u>Addition Vocabulary</u> **+** **Add** **Addition** **Plus** **And** **Count on** **More** **Sum** **Total** **Altogether** **Increase** **'Equals' or 'is the same as'**	**Ideas and mental strategies that children should master to enable them to succeed in written calculation. These are taught as the child becomes ready and may not be taught in this order:-** • Addition can be done in any order. e.g. 34 + 56 = 90 or 56 + 34 = 90 • When adding numbers we usually start with the largest number (so that you have to do less counting and so there is less potential for mistakes.) e.g. 27 + 5 = 32 • Children MUST:- o Know that '=' is an equals sign that means 'is the same as' o Know number bonds to 10 e.g. 1+9=10, 2+8= 10, 3+7=10, 4+6=10, 5+5=10 etc. o Know addition facts for all single-digit numbers. e.g. 1+1=2, 1+2=3, 1+4=5, 2+1=3, 2+2=4, 2+3=5 etc. o Know how to count forward in steps of 1,2, 5, 10 and 100 along a number line.  o Understand the number line as a continuum. A number line is just a tool that helps us count forwards and backwards – it has no 'official' starting or ending point. o Know how to use concrete apparatus available. e.g. using objects like multilink, Base 10, toys, blocks, Numicon o Understand place value. e.g. Knows that in the number 327, the digit '3' means '3 hundreds', the digit '2' means '20' and the digit '7' means 7 o Be able to partition numbers. e.g. Can split a number like 327 into 300 + 20 + 7 o Be able to count forwards and backwards in steps of different sizes. e.g. counting forwards in 1s – 1, 2, 3, 4, 5 etc; or in steps of 2 – 2, 4, 6, 8, 10 etc; or in steps of 5 – 5, 10, 15, 20, 25 etc. ; or in steps of 10 – 10, 20, 30, 40, 50 etc.. o Know doubles of numbers from 1-10 e.g. double 3 is 6, (or 2 lots of 3 is 6, or 2 times 3 is 3, or 2 groups of 3 is 6) o Known doubles of numbers from 10-20. e.g. double 12 is 24, (or 2 lots of 12 is 24, or 2 times 12 is 24, or 2 groups of 12 is 24) o Know that adding numbers always produces a larger answer.

Progression in Addition

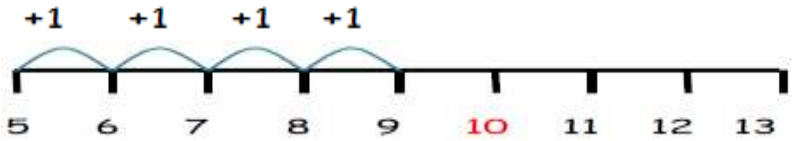
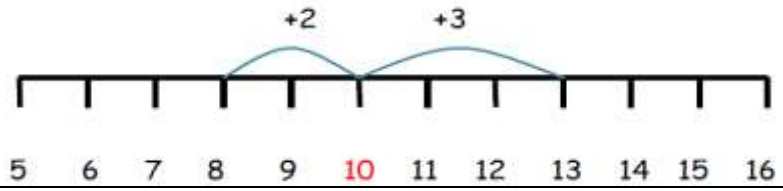
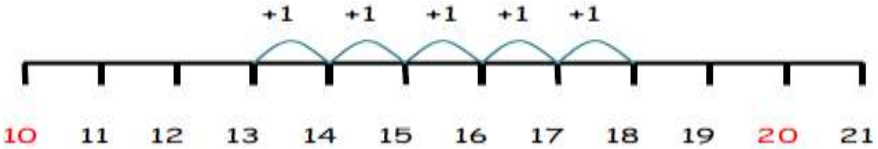
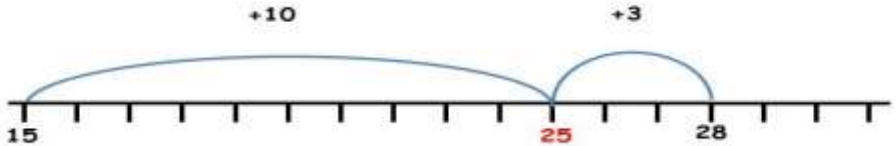
- Count up to 10 objects reliably.
- Find 'one more' than a number. e.g. when given a number, say 13, they can count on to find 'one more' e.g. 14.
- Add two or more groups of objects together to find a total of less than 10. These may be concrete apparatus or pictures.

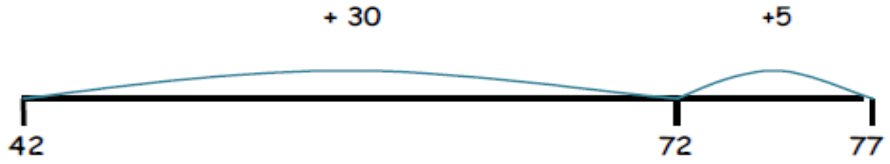
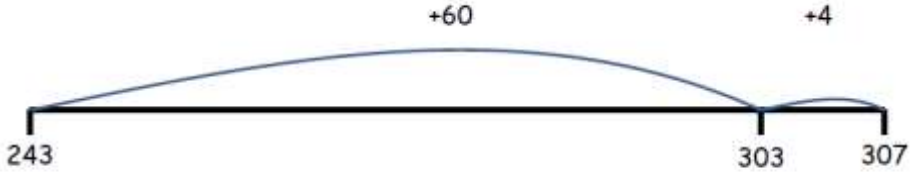
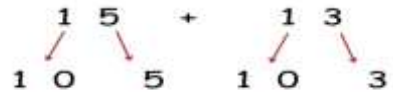


+ = 5

- Use the + and = signs to record mental calculations in a number sentence. e.g. 2+6=8

Non-Standard Methods

• **Count along a number line to add single digit numbers together to find a total of less than 10 e.g. 5 + 4 = 9**	
• **Add single digit numbers that bridge to 10 using a number line. This involves partitioning the smaller number in to 2 parts, one of which will add to the larger number to make 10 e.g. 8 + 5 = 13**	
• **Add a 2-digit number and a single digit number using a number line e.g. 13 + 5 = 18**	
• **Add two 2-digit numbers bridging through 10 using a number line. This involves partitioning the smaller number into 2 or more parts using knowledge of multiples of 2, 5 or 10. For example 15 + 13 = 15 + 10 + 3. If the tens digit is greater than 1, this can involve multiple jumps of 10.**	

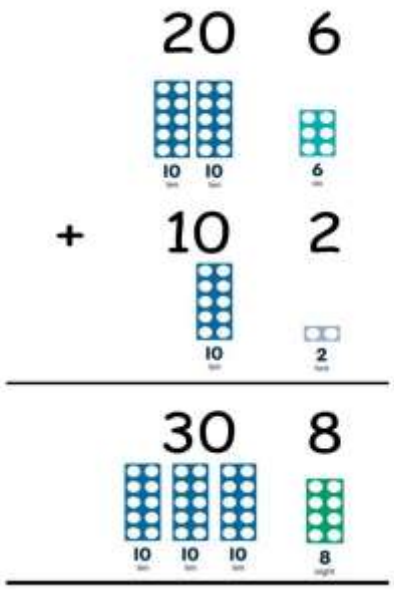
• **Add two 2-digit numbers adding the most significant digit first using a blank number line e.g. 42 + 35= 77**	
• **Add a 3-digit number and a 2-digit number using a number line e.g. 243 +64**	
• **Partition and recombine e.g. 15 +13= 28**	 Then 10+10 = 20 and 5+3 = 8 So 20+8 = 28
• **Partition in columns (Expanded Addition Method without carry figure) e.g., 24 + 15 =39**	24 + 15 = 2 0 + 4 1 0 + 5 3 0 + 9 = 3 9
• **Partition in columns (Expanded Addition Method with column sum greater than 10) e.g., 26 + 19 = 45** **N.B. The term carrying tens should be avoided at this stage and only introduced with standard written columnar method to avoid confusion.**	26 + 19 = 2 0 + 4 1 0 + 5 3 0 + 9 = 3 9

Standard Written Methods (Columnar Addition)

Additon without carrying using Numicon

Step 1: Add together the digits in the units column (6 + 2 = 8) and put the answer in the units column of the answer line

Step 2: Add together the digits in the tens column (2 + 1 = 3 actually this is twenty plus ten). Put the answer in the tens column of the answer line

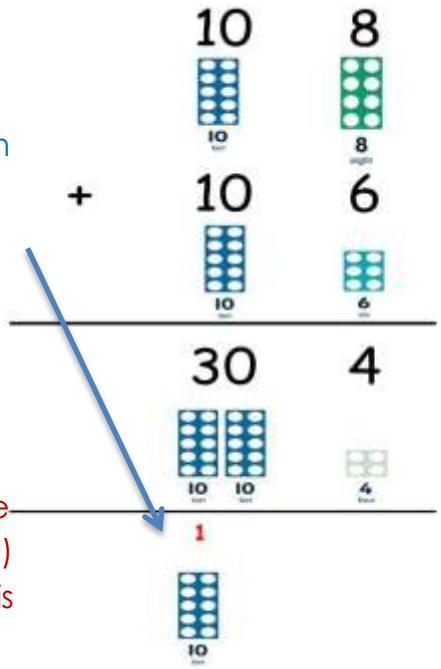


Addition with a carry figure using Numicon

Step 1: Add together the digits in the units column (6 + 8 = 14)

Step 2: As the number is more than ten, the units digit is put in the answer line and the extra ten is carried across. This is put underneath the answer line in the tens column.

Step 3: Add together the digits in the units column including the one that was carried across (1+1+1 = 3) or actually three tens or 30 which is put in the tens column of the answer line



2 digit add 1 digit

$$\begin{array}{r} 24 \\ +\ 5 \\ \hline 29 \\ \hline \end{array}$$

3 digit add 1 digit (with carry figure)

$$\begin{array}{r} 247 \\ +\ 6 \\ \hline 253 \\ \hline \end{array}$$

1

2 digit add 2 digit (with carry figure)

$$\begin{array}{r} 26 \\ +\ 38 \\ \hline 64 \\ \hline \end{array}$$

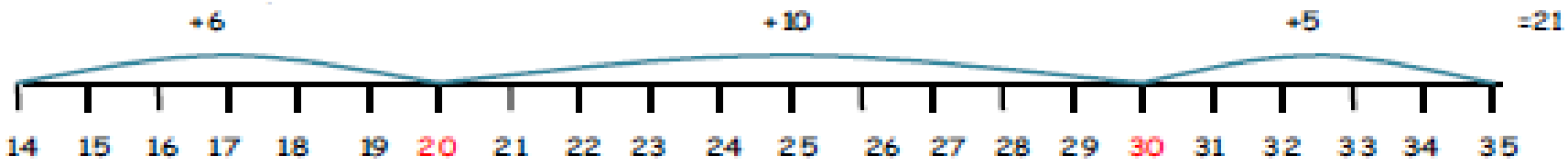
1

3 digit add 2 digit (with carry figure)	3 digit add 3 digit (with carry figure)	Adding decimals (with carry figure)
$\begin{array}{r} 129 \\ +\ \ 42 \\ \hline 171 \\ \hline 1 \end{array}$	$\begin{array}{r} 368 \\ +\ 256 \\ \hline 624 \\ \hline 1\ \ 1 \end{array}$	$\begin{array}{r} 126.41 \\ +\ \ 36.82 \\ \hline 163.23 \\ \hline 1\ \ 1 \end{array}$

<u>Subtraction Vocabulary</u>
 	
-
Subtract
Take Away
Minus
Less
Fewer
Difference
'Equals' or 'is the same as'
	Ideas and mental strategies that children should master to enable them to succeed in written calculation. These are taught as the child becomes ready and may not be taught in this order:-
	
	• Subtraction can be seen in two ways, either as **'taking away'** or as **'finding the difference'**
	• **'Taking Away'** is usually when subtracting a smaller number form a larger one, often a single digit from a 2-digit number e.g., 32 – 6 = 26.
	• **'Finding the difference'** has more explicit links to the complex subtraction used further in the school.
	• Children MUST:-
	  o Know that '=' is an equals sign that means 'is the same as'
	  o Know number bonds to 10 and the reverse. e.g. 1+9=10, 2+8+10, 3+7=10 etc and 10-1=9, 10-2=8, 10-3=7 etc
	  o Know number bonds to 100 (sometimes called **complements** to 100) e.g. . 20+80 = 100, 45+55=100, 100-43=57, etc.
	  o Understand the number line as a continuum. A number line is just a tool that helps us count forwards and backwards. It has no 'official' starting or ending point.
	  o Know that subtraction **cannot** be calculated in any order. e.g. 9-4=5 is not the same as 4-9 = -5
	  o Understand place value. e.g. know that in the number 327, the digit '3' means '3 hundreds', the digit '2' means '20' and the digit '7' means

Progression in Subtraction

- Use concrete apparatus to physically 'take away' from numbers less than 10.
- 'Find the difference' between two numbers by counting on. This can be using a number line, fingers or other apparatus.
- 'Find the difference' between smaller numbers by mentally counting on from a smaller number to a larger one. For example 35 – 14 using a number line. Start with 14 and count on to 35. The difference between the two is the answer.



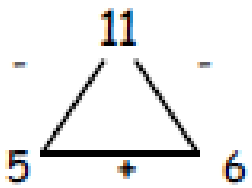
- Use concrete objects, pictures or apparatus to either 'take away' or 'find the difference' between two groups e.g., 8 – 3 = 5.

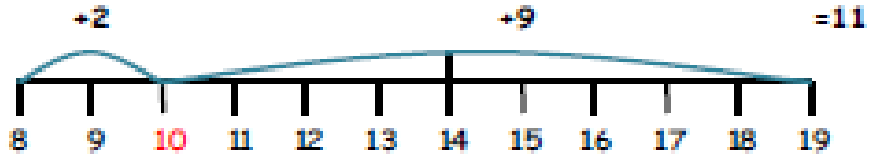
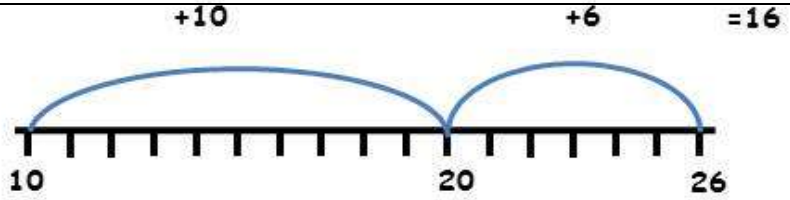
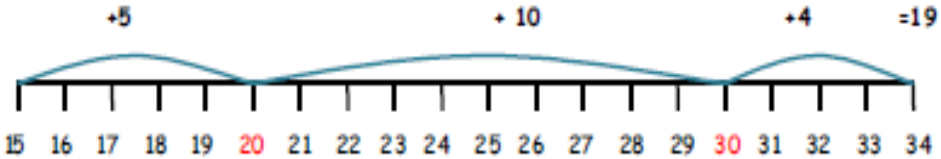
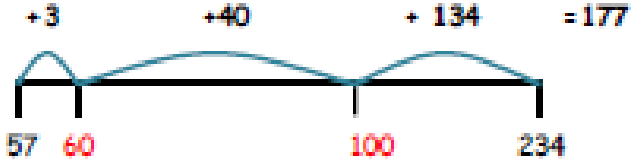
- Count on and back in 1s and 10's on a number line
- Count forwards and backwards in steps of different sizes e.g., counting in 1's, 2's, 5's and 10's from any given starting point
- Finding 'one less than' a number
- Use – and = signs to record mental calculations in number sentences e.g., 23 – 6 = 17
- Use addition/subtraction trios to understand subtraction and the inverse of addition.

11 – 6 = 5

11 – 5 = 6

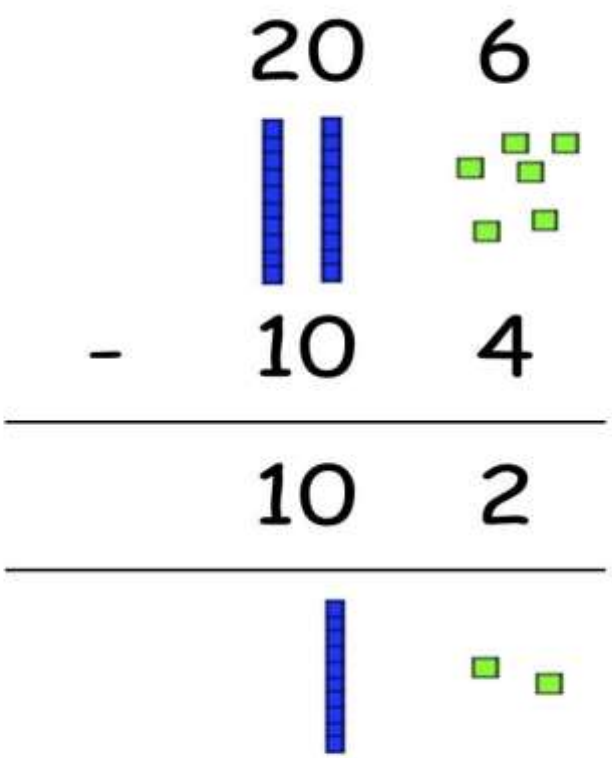
5 + 6 = 11



Non-Standard Methods	
• **Use 'counting on' with a number line from a single digit to a two digit number less than 20 e.g., 19 – 8 = 11**	
• **Subtract 10 from a two digit number using 'counting on' on a number line e.g., 26 – 10 = 16**	
• **Use a number line for 2-digit numbers subtract 2-digit numbers when 'bridging to ten' e.g., 34 -15 = 19**	
• **Use a number line for 2-digit numbers subtract 2-digit numbers using more efficient bigger jumps e.g., 76 – 29 = 47**	
• **Use a number line for 3-digit numbers subtract 2-digit numbers using efficient bigger jumps e.g. 234 – 57 = 177**	

Standard Written Methods (Columnar Subtraction)

Base 10 to set out column subtraction for 2-digit subtract 2-digit without 'exchanging'



Remember, the smallest digit always goes on the bottom

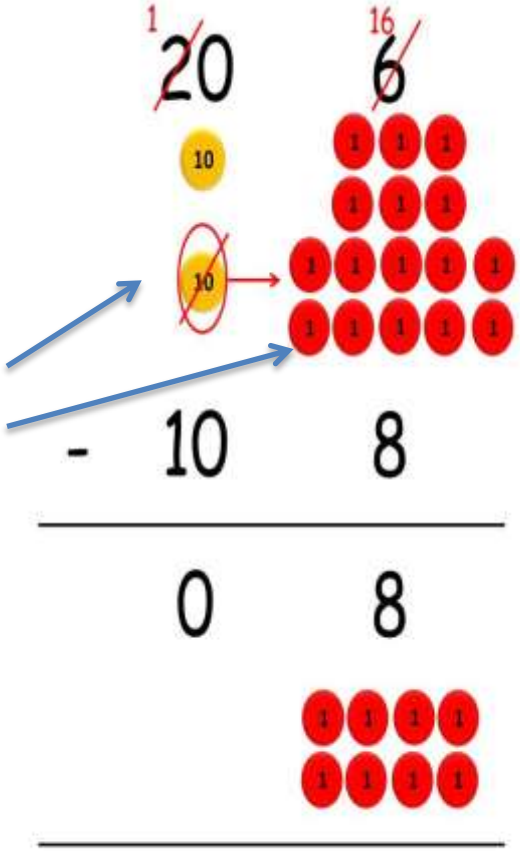
Place Value Counters to set out column subtraction for 2-digit subtract 2-digit with 'exchanging' and 're-grouping'

Step 1: The bottom digit must always be subtracted from the top digit BUT 6 – 8 will not work

Step 2: One of the tens counters from the top number is **exchanged** for 10 ones and these are **re-grouped** in the units column to give 16 ones (or 16)

Step 1: The unit column mini-calculation is now 16 – 8 = 8.

Step 4: As one of the tens has been exchanged the tens column mini-calculation is now 1 – 1 (actually 10 -10) = 0. This zero is included only to make clear to the pupil what has happened



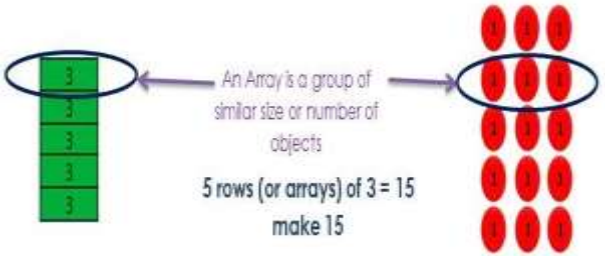
The bottom digit always as to be subtracted from the top digit (as this is the smaller number). In order for this to happen, one ten is exchanged from the tens column and converted into ten ones in the ones column.

Use columnar subtraction for 2-digit $\begin{array}{r} {}^{2}\cancel{3}\ \ {}^{14}\cancel{4} \\ -\ \ 2\ \ 8 \\ \hline 7 \\ \hline \end{array}$ **Exchange** one of the tens from the 30 to leave 20 in the tens column and **re-group** that 10 with the ones (or units) to make 14 in that column so you can now take away 8	**Use columnar subtraction for 3-digit numbers subtract 2-digit numbers** $\begin{array}{r} {}^{4}\cancel{5}\ \ {}^{14}\cancel{4}\ \ 6 \\ -\ \ 5\ \ 5 \\ \hline 4\ \ 9\ \ 1 \\ \hline \end{array}$
Use columnar subtraction for decimal numbers. $\begin{array}{r} 5\ \cancel{4}^{3}\ .\ \cancel{6}^{16} \\ -\ 2\ 2\ .\ 8 \\ \hline 3\ 1\ .\ 8 \\ \hline \end{array}$ Remember to put the decimal point in your answer space first to line up the columns.	**Subtraction of decimals where the numbers are expressed with different decimal places (e.g., 54.23 – 6.1)** $\begin{array}{r} {}^{4}\cancel{5}\ \cancel{4}^{14}\ .\ 2\ 3 \\ -\ \ 6\ .\ 1\ 0 \\ \hline 4\ 8\ .\ 1\ 3 \\ \hline \end{array}$ Use a zero as a place holder to make sure that the digits are lined up in the correct columns

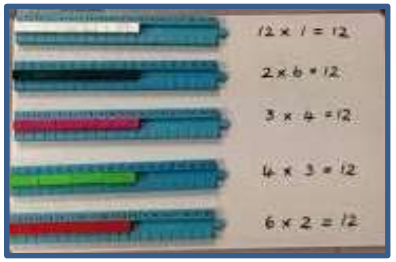
<u>Multiplication Vocabulary</u> **x** **Lots of** **Groups of** **Times tables** **Multiply** **Multiplication** **Multiple** **Product** **Array** **Double** **Columns** **Rows** **Repeated addition** **'Equals' or 'is the same as'**	**Ideas and mental strategies that children should master to enable them to succeed in written calculation. These are taught as the child becomes ready and may not be taught in this order:-** • Children MUST:- o Know that '=' is an equals sign that means 'is the same as' o Be able to recognise simple sequences of numbers e.g., 5, 10, 15, 20 (add five each time or count in fives); 2, 4, 6, 8 (add 2 each time r count in 2's) o Be able to use a method for addition and subtraction (see previous sections) o Know that multiplication can be calculated in any order (3 x 4 = 12 and 4 x 3 = 12) o Be able to show multiplication facts using **arrays** (of rows and columns)  **2 rows of 4 are 8** **2 x 4 = 8**  **2 columns of 3 are 6** **3 x 2 = 6** o Know that multiplication and division are the inverse of each other Understand **place value** e.g., know that in the number 327, the digit '3' means 3 hundreds (300), the digit 2 means two tens (20) and the digit 7 means seven ones (7) o Be able to **double** and **halve** numbers from 1 to 100 e.g., Double 4 is 4 x 3 = 8 and half of 8 is 8 ÷ 2 = 4. o Multiplication is **repeated addition** e.g., to find 4 x 3, you add 4 groups of 3 or you add 3 four times (3+3+3+3).

Progression in Multiplication

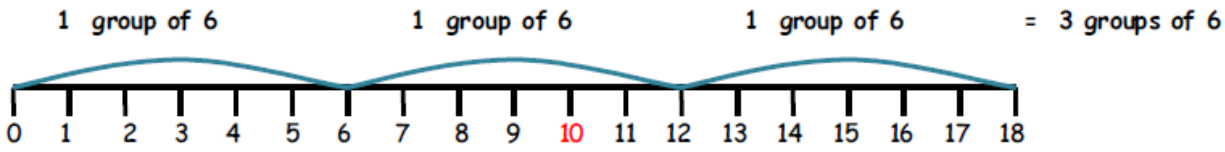
- Be able to put objects into groups of the same number
- Use practical apparatus to represent arrays of a multiples or factors



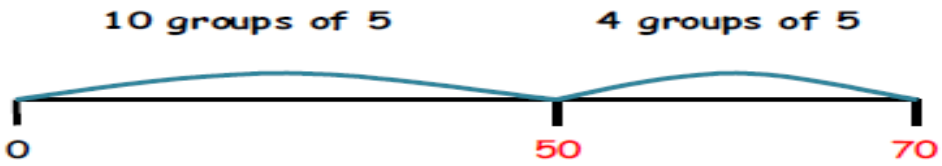
- Be able to use a number track and Numicon for repeated addition



- Use a number line and record the jumps (how many groups of…) for single digit times single digit calculations e.g., 3 x 6 = 18

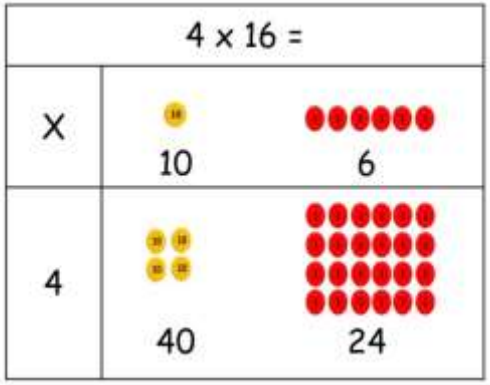


- Use a number line for 2-digit numbers times single digit numbers e.g., 24 x 8 =



Non-Standard Methods

- **Use concrete apparatus to set out in Grid Method for 2-digit number times single digit number**
  e.g., 3 x 16 = 48



By partitioning the, 2-digit number, we can work out the value of the mini-calculation in each column by repeated adding of ten or by times tables facts. The final answer is gained when these two values are added together:-

40 + 24 = 64

- **Use Grid Method for 2-digit numbers times single digit numbers (without additional apparatus)**
  e.g., 24 x 3 = 72

x	20	4
3	60	12

$$\begin{array}{r} 60 \\ +\ 12 \\ \hline 72 \end{array}$$

- **Use Grid Method for 2-digit numbers times 2-digit numbers (practical apparatus could be used to support if needed)**
  e.g., 24 x 32 = 768

x	20	4
30	600	120
2	40	8

$$\begin{array}{r} 600 \\ 120 \\ 40 \\ +\ \ \ 8 \\ \hline 768 \end{array}$$

- **Use Grid Method for 3-digit numbers times 2-digit numbers**

x	200	40	7
20	4000	800	140
4	800	160	28
+	4800	960	168

```
  4800
   960
+  168
  5928
  1 1
```

- **Use Grid Method for 3-digit numbers times 3-digit numbers**

x	300	20	1
100	30000	2000	100
10	3000	200	10
8	2400	160	8
+	35400	2360	118

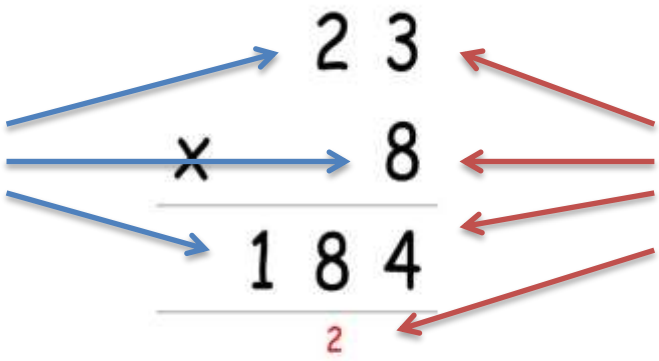
```
  35400
   2360
+   118
  37878
```

Standard Written Methods

Short Multiplication for 2-digit numbers times single digit numbers e.g., 23 x 8 =

Step 2

8 x 2 *(actually two tens)* gives 16. Then add the 2 *(also worth two tens)* to make a final total of 18 *(actually 18 tens or 180)*

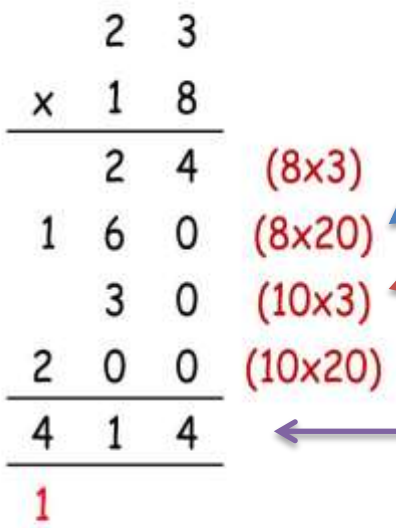


Step 1

8 x 3 = 24

Put the 4 in the ones *(or units)* column and carry the 2 *(actually two tens)*, placing the digit under the tens column

Expanded Multiplication for 2-digit times 2-digit numbers e.g., 23 x 18 =



Step 1: Multiply both of the digits in the larger number by 8, recording each answer (8x3 = 24) and (8x20 = 160) on separate lines.

Step 2: Multiply both of the digits in the larger number by 10, recording each answer 10x3 = 30) and (10x20 = 200) on separate lines.

Step 3: Add all of the resulting numbers together. The sum is the answer to the calculation

Then use Expanded Multiplication for 3-digit times 2-digit numbers (e.g., 234 x 64 =) and also for 3-digit times 3-digit (e.g., 234 x 456 =) numbers.

Long Multiplication for 2-digit times 2-digit numbers (e.g., 23 x 18 =)

Step 1: 8 x 3 = 24. Write the 4 in the ones (or units) column and the 2 (really two tens) as a carry figure in the tens column

Step 4: 1 x 3 (really 3 times 10) = 3 (really 30). Write 3 in the tens column

Step 2: 8 x 2 (really 8 times 20) = 16. Add the two tens carried over makes 18. Write the 8 in the tens column and the 1 (really ten times tens) in the hundreds column

Step 5: 1 x 2 (really 10 times 20) = 2 (really 200). Write 2 in the hundreds column

Step 3: Place a '0' in the units column as all the calculations are now multiples of 10

Step 6: Add up the sum of the 'mini-calculations' and write it in the bottom answer row.

		2	3
	×	1	8
	1	8 ~~2~~	4
	2	3	0
	4	1	4
1			

Then use long multiplication with 3-digit numbers times 2-digit numbers and also using decimal numbers to 2 decimal places.

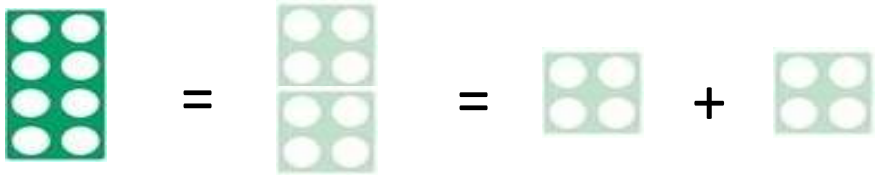
<u>Division Vocabulary</u> **÷** **Lots of** **Groups of** **Groups** **Jumps** **Equal** **Halve** **Divide** **Division** **Divided by** **Remainder** **Factor** **Decimal** **Decimal Place** **Divisible** **'Equals' or 'is the same as'** **Quotient**	**Ideas and mental strategies that children should master to enable them to succeed in written calculation. These are taught as the child becomes ready and may not be taught in this order:-** • Children MUST:- o Know that '=' is an equals sign that means 'is the same as' o Be able to put objects into groups of the same number o Be able to recognise simple sequences of numbers e.g., 5, 10, 15, 20 (add five each time or count in fives); 2, 4, 6, 8 (add 2 each time r count in 2's) o Be able to use a method for adding and subtracting (see previous sections) o Recall multiplication facts to 12 x 12 and derive division facts e.g., 5 x 4 = 20, so 20 ÷ 5 = 4 and 20 ÷ 4 = 5 o Know that multiplication and division are the inverse of each other e.g., 2 x 6 =12 and 12 ÷ 6 = 2 o Be able to find **half (½)** and **a quarter (¼)** of a group of objects or a whole number. o Understand **place value** e.g., know that in the number 327, the '3' means 3 hundreds (300), the 2 means two tens (20) and the 7 means seven ones (7) o Be able to double and halve numbers from 1 to 100 e.g., Double 4 is 8 (4 x 2 = 8); half of 8 is 4 (8 ÷ 2 = 4) o Know that division **cannot** be calculated in any order e.g., 12 ÷ 4 = 3 is not the same as 12 ÷ 3 = 4

Progression in Division

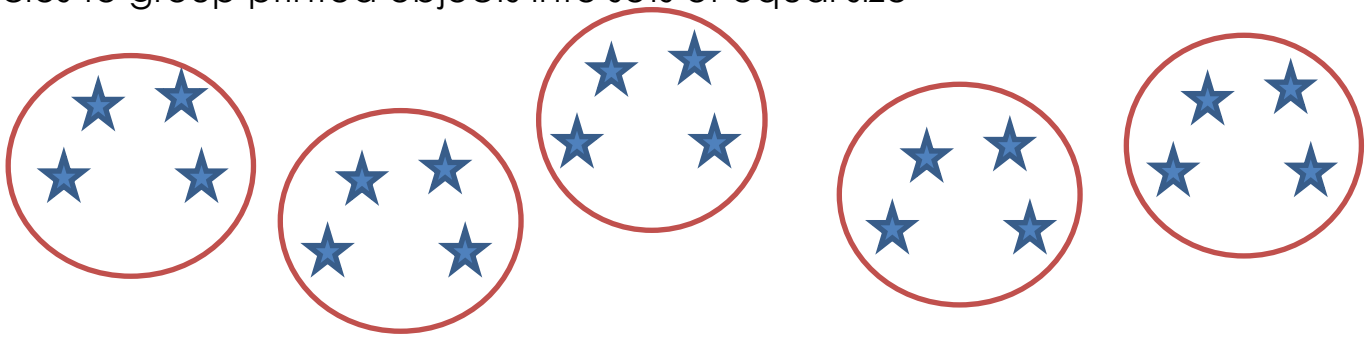
- Share objects into groups of equal size

8 objects shared into four groups of equal size

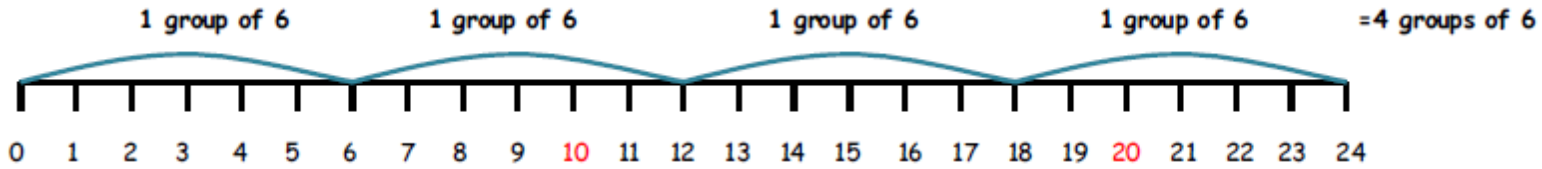
- Use Numicon to divide numbers into chunks of equal size

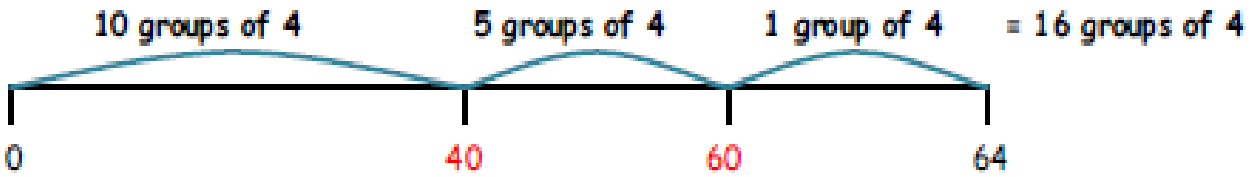
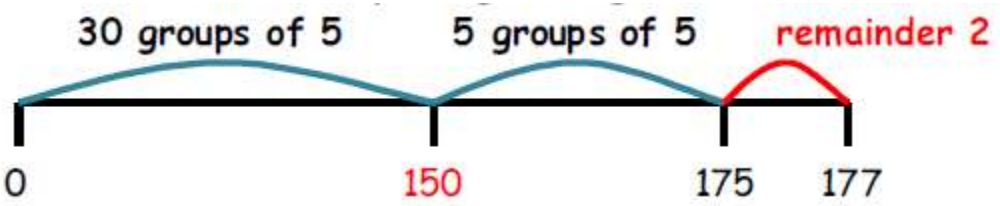


- Use circles to group printed objects into sets of equal size



- Repeated addition using a number line



Non-Standard Methods
Use number line chunking for 2-digit numbers divided by single digit numbers e.g., 64 ÷ 4 = 16 Step 1: Write down the times tables facts for the divisor '4' Step 2: Draw a number line starting at 0 and ending with the 'target number' e.g., 64 Step 3: Choose a multiple of 4 that is closer to the target number (10 x 4 = 40 or ten groups of 4) Step 4: Record the jump on the number line and work out what the difference between that number and the target number. (64 – 40 = 26) Step 5: Repeat with one or more jumps until you reach the target number. Step 6: Add up the total number of groups you have jumped and this is the answer.   
Use number line chunking for 3-digit numbers divided by 2-digit numbers with remainders and using more efficient jumps e.g., 177 ÷ 5 = 35 r2      **Also use number line chunking for 3-digit numbers divided by 2-digit numbers with remainders**

Use column chunking for 2-digit numbers divided by single digit numbers e.g., 64 ÷ 4 = 16

Step 1: Write down the times tables facts for the divisor '4'

Step 2: Write the largest multiple of 10, 5 or 2 (in this case 4 x10 = 40) that you can subtract from the target number and carry out the mini calculation (64 – 40 = 24)

Step 3: Write the new target number down (24) and repeat the previous steps until You are left with 0

Step 4: Count up then total number of groups that you have subtracted from the Target number. This is the answer.

	6	4	
-	4	0	(10 groups of 4)
	2	4	
-	2	0	(5 groups of 4)
		4	
-		4	(1 group of 4)
		0	

Total groups of 4 = 10 + 5 + 1 = 16

Use column chunking for 3-digit numbers divided by 2-digit numbers with remainders and using more efficient jumps e.g., 177 ÷ 5 = 35 r2

Step 1: Write down the a multiple of 10 for the divisor '5' that is near to the target number and carry out the mini calculation (177 – 150 = 27)

Step 2: Write down the new target number (27) and write down a times table fact for the divisor 5 that is nearest to the new target number (25)

Step 3: Carry out the mini-calculation (27 – 25) and write down the remainder.

	1	7	7	
-	1	5	0	(30 groups of 5)
		2	7	
-		2	5	(5 groups of 5)
Remainder			2	

Total groups of 5 = 30 + 5 = 35 r 2

Also column chunking for 3-digit numbers divided by 2-digit numbers with remainders

Standard Written Methods

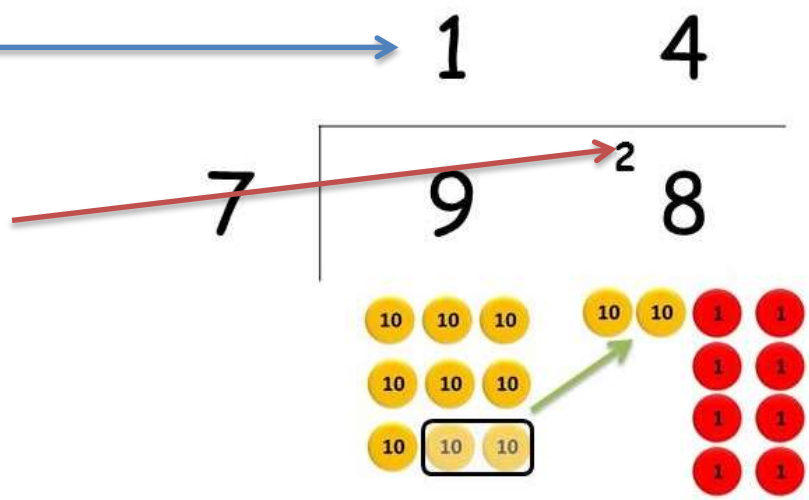
Use short division method to divide a 2-digit number by a single digit number e.g., 98 ÷ 7 =

Step 1: How many groups of 7 are there in 9?
Write the number 1 above the line.

Step 2: How many are left over? Carry the digit 2
to the next column. The new value of this column is 20 + 8 = 28

Step 3: How many groups of 7 are there in 28?
Write the digit 4 above the line.

Step 4: There are no remainders so the answer is 14.

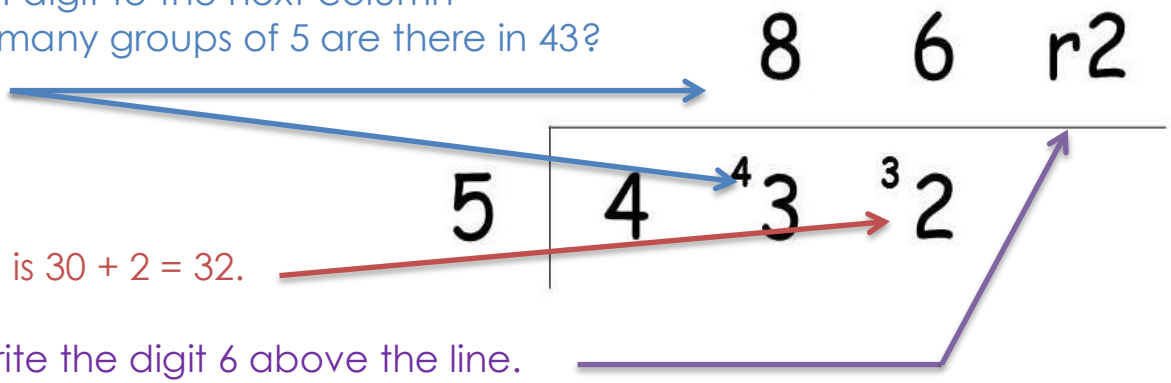


Use short division method to divide a 3-digit number by a single digit number with remainders e.g., 432 ÷ 5 =

Step 1: There are no groups of 8 in 4 so carry the 4 digit to the next column
The new value of this column is 40 + 3 = 43. How many groups of 5 are there in 43?
Write the digit 8 above the line.

Step 2: How many are left over? Carry the digit 3
to the next column. The new value of this column is 30 + 2 = 32.

Step 3: How many groups of 5 are there in 32? Write the digit 6 above the line.
There is a remainder of 2, write this at the side of the answer line.



Then use short division to divide larger numbers and also decimal numbers to 2 decimal places by single digits

Use long division method to divide a 3-digit number by a two digit number without remainders

Step 1: Find the nearest multiple of 10x24 to the target number 864. This is 720 (30x24)
Put a digit '3' in the tens column.

Step 2: The difference between 864 and 360 is 144. This is the new target number.
How many multiples of 24 are there in 144? The nearest multiple of 24 is 144 (6x24).
Put a digit '6' in the units column.

Step 3: The difference between 6x24 and 144 is 0. Therefore there is no remainders and
The answer is 36.

```
        3 6
2 4 | 8 6 4
    - 7 2 0  (30x24)
      1 4 4
    - 1 4 4  (6x24)
          0
```

Use long division method to divide a 3-digit number by a two digit number with remainders

Step 1: Find the nearest multiple of 10x15 to the target number 432. This is 300 (20x15)
Put a digit '2' in the tens column.

Step 2: The difference between 432 and 300 is 132. This is the new target number.
How many multiples of 15 are there in 132? The nearest multiple of 15 is 120 (8x15).
Put a digit '8' in the units column

Step 3: The difference between 132 and 120 is 12. There are no groups of 15 in 12
This is therefore a remainder and is written at the side of the answer line.

```
        2 8 r 1 2
1 5 | 4 3 2
    - 3 0 0  (20x15)
      1 3 2
    - 1 2 0  (8x15)
        1 2
```

Year Group Expectations

In Reception (EYFS), children will:

- add, subtract, multiply and divide using practical equipment.

In Year One, children will:

- continue to use practical equipment and pictorial representations of objects.
- begin to record their work in number sentences that may involve filling in the missing number.
- mentally recall:
    - Number bonds
    - Addition and subtraction facts
    - Odd and even numbers
    - Knowledge of facts e.g. 10 more/10 less

In Year Two children will:

- know addition and subtraction facts to 20
- count a range of objects
- use number facts to 20 to solve practical problems
- record their work in number sentences in the following order:-
    - 1 digit + 1 digit (e.g. 4+2=)
    - Swap over (e.g. 2+4=)

    - 2 digit + 1 digit (12+4)
    - Add 10 to a number
    - Add 9 and 11 to a number by adding on 10 then adjusting by 1
    - 2 digits + 1 digit (36+4=)
    - 2 digits add tens (13 + 10=)
    - 2 digits + teens (44 + 13 =)
    - 2 digit + 2 digit (22 + 34 =)
- Multiply and divide numbers in the following ways:-
    - Using practical equipment
    - Rote counting
    - Knowing odd and even numbers
    - Chanting times tables
    - Doubling and halving numbers up to 20
    - Mental recall of 2x 5x 10x tables
    - Using arrays to understand that 2x5 and 5x2 give the same answer
    - Writing number sentences
    - Using multiplication facts to solve problems
    - Knowing that times and divide are the inverse of each other

In Year Three children will:

- begin adding numbers by partitioning and recombining
    - two digit and two digit numbers
    - two digit and three digit numbers
- Begin subtracting numbers by partitioning
    - two digit from two digit numbers
    - two digit numbers from three digit numbers

    - three digit from three digit numbers)
- use practical apparatus to help them add and subtract
- continue to practice Y2 multiplication and division activities.
- recall multiplication facts for 2x 5x 10x tables
- recall multiplication facts for 3x 4x tables
- Use multiplication facts to solve problems and calculate mathematical statements.
- Understand that multiplication can be done in any order
- use their knowledge of partitioning numbers to record their work using a written method
- use partitioning and the grid method to record multiplication calculations for 2-digit times 1-digit numbers
- begin to know the division facts firstly for the 2x, 3x, 4x, 5x, 8x and 10x tables then eventually up to 12x12
- use practical apparatus to group and share
- begin to record multiplication and division calculations using written methods

In Year Four children will:

- use partitioning to add and subtract larger numbers up to four digits
- use standard written methods (exchange method or decomposition) to subtract
- recognise and use multiplication and division facts in calculating
- continue to partition numbers and use grid method for multiplication for 2-digit times 2-digit numbers
- begin to multiply numbers by 10, 100 and 1000
- learn to move the digits left and use 0 as a place holder when multiplying by 10, 100 and 1000

In Year Five children will:

- use efficient standard written methods for columnar addition and subtraction (using practical apparatus where needed) with numbers of more than four digits
- use decomposition in subtraction with increasingly large numbers

- add and subtract decimal numbers (to 2 d.p) including money
- know all their multiplication and division facts up to 12x12 and recall them quickly and accurately
- know and use the mathematical terms: multiple; factor and common factor
- know and use squared numbers and cubed numbers and the related notation (squared^2 cubed^3)
- know that a prime number has only two factors, 1 and itself
- recognise the prime numbers quickly e.g., 1, 3, 5, 7, 11, 13, 17, 19….
- begin to use an efficient standard written method for multiplication (using practical apparatus to support if needed)
- begin to use short division and long division as standard written forms (using practical apparatus to support if needed)
- use their knowledge of calculation strategies to solve problems and puzzles of increasing complexity

In Year Six children will:

- use efficient standard written methods for all four operations with increasingly large numbers
- use their knowledge of factors and multiples effectively in multiplication e.g., 43 x 6 is double 43 x 3 OR 28 x 50 is half of 28 x 100
- use their knowledge of calculation to solve problems and puzzles containing whole numbers and decimals and in application to calculation problems using algebra, fractions, ratio and proportion
- know that dividing whole numbers and decimals by up to 10,000 is the opposite of multiplication and therefore the digits move to the right.

Additional Support for Children

It is expected that the majority of children will progress through the year groups with the ability to demonstrate that they can not only retain the calculation skills they have been taught but can also apply them in different contexts (e.g., problem solving in maths and science investigations). However, there will be occasions when some children need support with specific aspects of calculation to ensure that they make good progress in maths.

First Class@Number and **Success@Arithmetic** are intervention programmes that the school has adopted to ensure that this support is available for children. They are both very practical programmes based on the use of concrete apparatus to foster secure understanding and reinforce skills where the child is not as confident. Each group will follow a programme that lasts approximately one half term with an initial diagnostic assessment and a progress assessment at the end of the intervention. They are delivered by support staff who have been trained in the programme contents and the equipment used. Ms Howard will then oversee the delivery and impact of the intervention in discussion with the class teacher.

Half termly evaluations of progress using the assessment tracking booklets (or through formative tests) are used to identify children for whom this support will be appropriate. The tracking booklets relate to the national curriculum skills in maths that are appropriate for each year group. Gaps in number and calculation skills can then be addressed quickly and effectively.